Solutions 1D

1. a) $x = 4/3$, b) $x = -9$, c) $x = -3$ or $x = -1$ **each 1 p**

2. Denote t the time in some units (e.g. minute), v the velocity of tortoise in meters per time unit and s_0 the head start (m).

Assume that Achilles reaches the tortoise at time t, then

$$10vt = s_0 + vt \Leftrightarrow t = \frac{s_0}{9v}$$ **1 p**

The distance s Achilles reaches the tortoise should be smaller than 5000 m.

$$10v\left(\frac{s_0}{9v}\right) = \frac{10}{9}s_0 < 5000$$ **1 p**

hence

$$s_0 < \frac{9}{10}\cdot 5000 = 4500\,(\mathrm{m})$$ **1 p**

Other correct and justified solutions are also accepted.

3. Correct model e.g.

$$\frac{x}{\sin\beta} = \frac{d}{\sin\left(180^\circ - \alpha - \left(180^\circ - \beta\right)\right)} = \frac{d}{\sin(\beta - \alpha)}$$ **1.5 p**

Solution in symbolic form e.g.

$$x = \frac{d\sin\beta}{\sin(\beta - \alpha)}$$ **1 p**

Numerical solution $x = 762$ m **0.5 p**

4.

a) 70% · 90% = **63%** **1.5 p**

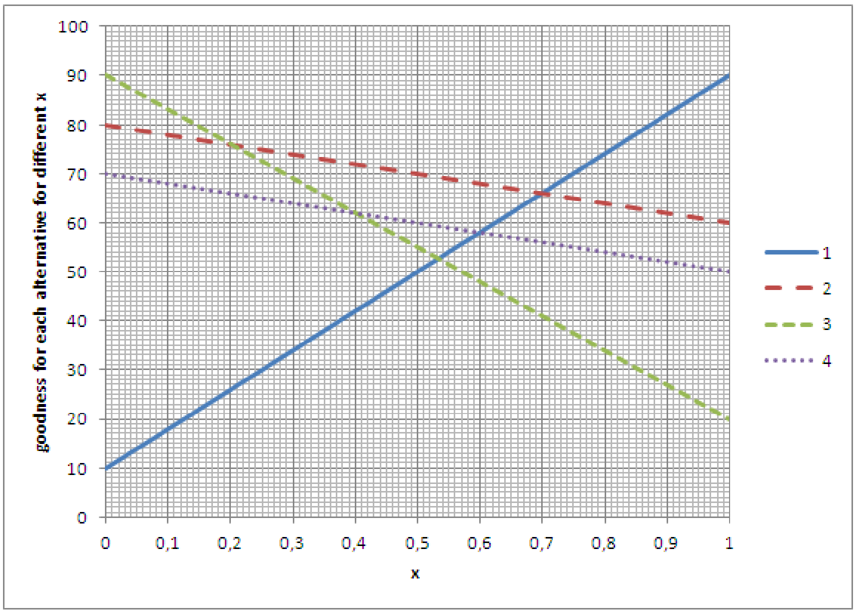
b) 70% · 90% + 30% · (100% - 80%) = **69%** **1.5 p**

5.

a) 1 **0.5 p**

b) 2 (justification: goodness values of each alternative calculated) **1 p**

c) 4 (justification e.g. by drawing the graphs of the goodness functions of each alternative) **1.5 p**



6. a) *ABC*+* **1 p**

   b) *AB*C+D** **2 p**

7. Item a) gives 1 point, because there is logical implication. Item b) gives 3 points.

8A. This is a 2D-motion task. It is generally known that horizontal velocity is constant and the ball will fall down with the same velocity as it was initially thrown. The initial speed can be calculated from horizontal distance travelled in time and the initial angle from trigonometry. Because the vertical velocity decreases, the calculated vertical velocity value will be smaller than the initial value.

$$\boldsymbol{v}_o = \boldsymbol{v}_{xo} + \boldsymbol{v}_{yo} \quad and \quad v_o^{\,2} = v_{xo}^{\,2} + v_{yo}^{\,2}$$

a) $v_{xo} = \frac{\Delta x}{\Delta t} = \frac{6.00\ m}{0.20\ s} = 30.0\ m/s$

The angle by trigonometry: $\frac{\Delta height}{\Delta x} = \tan\alpha \rightarrow \ arctan\frac{10.20}{6.00} = 59{,}5°$ → **Select 60°**

Initial speed $v_o = \frac{v_{xo}}{cos\alpha} = 59.2$ m/s → **Select 60 m/s** **Both correct equals 1 p**

b) The shape of the trajectory is parabolic, so the highest point is reached at the half of the time of flight. At the time 10.50 s the ball is almost back at ground, so the half is bigger than 5.25 s. **Select 5.30 s.**

Height on both 4.62 s and 10.50 s are definitely below the maximum.**Correct answer 1 p**

c) Vertical velocity is zero, but the horizontal velocity is constant, so **velocity2 = 30 m/s.**

Correct answer 1 p

Correct answers without any reasoning or calculations yields only 2 pts.

8B. $pH = -lg[H^+] \rightarrow [H^+] = 10^{-pH}$

a) $[H^+] = 10^{-3}$ mol/L = 0.001 mol/L

n_{HCl} = 0.001 mol/L * 1000 L = 1 mol

$V_{HCl} = \frac{n_{HCl}}{c_{HCl}} = \frac{1\text{ mol}}{1\text{ mol/L}} = \mathbf{1\ L}$ **Calculated correct answer 1p**

b) $[H^+] = 10^{-4}$ mol/L = 0.0001 mol/L.

Calculation as above or deduction gives $V_{HCl} = \frac{0.1\text{ mol}}{1\text{ mol/L}} = \mathbf{0.1\ L}$ **1 p**

c) Neutralization of approximately 1000 L of 0.010 mol/l KOH :

$n_{HCl} = n_{KOH}$ = 0.010 mol/L * 1000 L = 10 mol

$V_{HCl} = \frac{10\text{ mol}}{1\text{ mol/L}} = 10\text{ L}$

According to part b) one needs approximately 0.1 L of HCl to change pH to 4

→ total 10.1 L → **10 L**

Justified or calculated correct answer 1 p

9A.

a) $m_{He} = \rho_{He} \cdot V_{He} = 0{,}179\text{ kg/m}^3 \cdot 8.34 \cdot 10^3\text{ m}^3 = 1493\text{kg}$

$M_{tot} = m_{He} + m_{capsule} + m_{balloon} + m_{Felix} = (1493 + 1315 + 1682 + 200)\text{ kg} = 4690\text{ kg}$

Buoyancy: $N = g \cdot \rho_{Air} \cdot V_{balloon1} = 9.81\text{ m/s}^2 \cdot 1.2\text{ kg/m}^3 \cdot 8.34 \cdot 10^3\text{ m}^3 = 98178\text{ N}$

Sum of forces: $\sum F = N - M_{tot} \cdot g = (98178 - 46009)\text{N} = 52170\text{ N}$

Acceleration: $F = m \cdot a \Leftrightarrow a = \frac{\sum F}{M_{tot}} = \mathbf{11.1\ m/s^2.}$ **1 p**

b) At the jump altitude gravity and buoyancy are equal but opposite direction.

$N = g \cdot \rho_{Air} \cdot V_{balloon2} =$

$M_{tot} \cdot g \Leftrightarrow \rho_{Air} = \frac{M_{tot}}{V_{balloon2}} = 4960\text{ kg}/3.84 \cdot 10^5\text{ m}^3 = \mathbf{0.013\ kg/m^3}$ **1 p**

c) If we use constant acceleration, we can approximate. Time duration of deceleration is (4 min 22 s) – 38 s = 262 s – 38 s = 224 s. Speeds need to be converted 1343 km/h = 373 m/s, and 277 km/h = 77 m/s.

Average acceleration

$a = \frac{\Delta v}{\Delta t} = \frac{(77-373)m/s}{224\ s} = \mathbf{-1.3\ m/s^2}$ **1 p)**

9B. $M_{Ni} = 58.7$ and $M_{NiS} = 58.7 + 32.1 = 90.8$

$m_{ore} = 1000$ kg

Nickel sulfide mass in the ore $m_{NiS} = 0.0034 \bullet 1000 \text{ kg} = 3.4 \text{ kg}$ **1 p**

Nickel mass calculated with molar masses:

$$\frac{m_{Ni}}{M_{Ni}} = \frac{m_{NiS}}{M_{NiS}}$$ **1 p**

$$m_{Ni} = \frac{m_{NiS}}{M_{NiS}} \times M_{Ni}$$

$$m_{Ni} = \frac{3.4 \text{ kg}}{90.8} \times 58.7 = 15.94 \text{ kg} \approx \mathbf{16\ kg}$$ **1 p**

10A. $V = 0.5$ l => $m = 0.5$ kg

$$Q = m \cdot L_f = m \cdot c \cdot \Delta T,$$

in which L_f is heat of fusion of the transition from ice to water is 333 kJ/kg, and c is the specific heat of water, 4.19 kJ/(kg · °C).

$$m_{ice} = \frac{Q}{L_f} = \frac{m_{water} \cdot c \cdot \Delta T}{L_f} = \frac{0{,}5\, kg \cdot 4{,}19\ \frac{\text{kJ}}{\text{kg °C}} \cdot 4\text{°C}}{333\ \frac{\text{kJ}}{\text{kg}}} = \frac{8{,}38 \text{ kg}}{333} = 0{,}025 \text{ kg}$$

So you need 25 g of ice and then you need to have 3 ice cubes.

Either of the equations remembered 1 p, and the number of ice cubes correct 3 p.

10B. $C_4H_{10} + 6{,}5O_2 \rightarrow 4CO_2 + 5H_2O$ or $2C_4H_{10} + 13O_2 \rightarrow 8CO_2 + 10H_2O$ **1 p**

$$\frac{n_{CO_2}}{n_{C_4H_{10}}} = \frac{4}{1} \rightarrow n_{CO_2} = 4n_{C_2H_6}$$ **1 p**

$$\rightarrow V_{CO_2} = 4V_{C_4H_{10}} = 4 * 2m^3 = 8\ \boldsymbol{m^3}$$ **1 p**

One can also calculate: number of moles of butane (89.3 mol)
→ number of moles of carbon dioxide (357.1 mol)
→volume of carbon dioxide either using molar volume 22.4 L/mol (8000Ll)
or using ideal gas law (8 m^3).