Engineering Entrance Exam 1D, 2014 solutions

1. a) $\frac{24}{35}$, b) 4, c) $3q$ d) $6a^2$, e) $\frac{a\sqrt[3]{b}}{10}$, f) $s(5r-9s)$ or $5rs-9s^2$ **(each 0.5 p)**

2. $V=\pi r^2 h$ **(0.5 p)**

$A=2\pi rh+2\pi r^2$ **(0.5 p)**

$A=2\pi rh+2\pi r^2=2\pi r\frac{V}{\pi r^2}+2\pi r^2=\frac{2V}{r}+2\pi r^2$ **(1 p)**

r [m]	$A(r)=\frac{2V}{r}+2\pi r^2$ [m^2]
…	…
0.45	5.717…
0.55	5.537…
0.65	5.731…
…	…

Answer **0.55 m** **(1 p)**

Other justified solution methods are accepted.

3. Law of sines: $|AC|=\frac{|AB|\sin(\beta)}{\sin(180^\circ-\alpha-\beta)}$ **(1 p)**

$x=A_x+|AC|\cos(90^\circ-\alpha)$, $y=A_y+|AC|\sin(90^\circ-\alpha)$ **(1.5 p)**

$x\approx 3.6$, $y\approx 4.4$ **(0.5 p)**

Alternative solution: Model e.g.: equations of lines AC and BC:

$y=x\tan(90^\circ-\alpha)+A_y$, $y=x\tan(\beta-90^\circ)+B_y$ **(1.5 p)**

Intersection point:

$x=\frac{B_y-A_y}{\tan(90^\circ-\alpha)-\tan(\beta-90^\circ)}$, $y=x\tan(90^\circ-\alpha)+A_y$ **(1 p)**

$x\approx 3.6$, $y\approx 4.4$ **(0.5 p)**

Other justified solution methods are accepted.

4. a) $R(10)=e^{-0.2\cdot 10}10^2 \approx 13.53$ **(1 p)**

b) $A \approx 10\dfrac{R(10)+R(20)}{2}+10\dfrac{R(20)+R(30)}{2}+10\dfrac{R(30)+R(40)}{2}$

or directly using trapezoid rule:

$$A=10\left[\frac{1}{2}R(10)+R(20)+R(30)+\frac{1}{2}R(40)\right]$$ **(1 p)**

Correct solution $A\approx 10\left[\frac{1}{2}13.53+7.33+2.23+\frac{1}{2}0.54\right]=165.95\approx 166$ **(1 p)**

5. The result can be justified by the following model or other calculation.

	S	**-S**	
R	¼	0	¼
-R	¼	½	¾
	½	½	

S: storm, -S: no storm, R: red sky. –R: gray sky

a) 100% **(1.5 p)**
b) ¼ + ½ = ¾ = 75% **(1.5 p)**

6. The missing numbers are marked with red and bold.
a) 4 4 8 16 32 **64** **128** (Sum of all previous numbers) **(1p)**
b) 1. 4 1 4 **2 1** (Square root of 2) **(1p)**
c) 3. 1 4 1 **5 9** (π, with more decimals) **(1p)**

7. a) Twice, 2 times **(1.5 p)**
b) Station 8; 4 passes **(1.5 p)**

8A) You are back to the starting position. There are at least 3 methods to solve.

1. use the following equations in all four parts of the curve.

   $$x = x_0 + v_0\, t + \frac{1}{2}\, at^2$$

   $$a = \frac{v_1 - v_0}{\Delta t}$$

2. use average velocities multiplied the time in all four parts of the curve.
3. calculate the area between the velocity curve and x-axis

Grading:

Positive and negative velocity understood **(1p)**
Negative acceleration means slowing down and turning back **(1p)**
Correct answer, back at origin **(1p)**

8B) $m_{NaNO_2} = 880\ g$

$$M_{NaNO_2} = 69.0$$
$$M_{NaNO_3} = 85.0$$

Each NaNO$_3$ produces one NaNO$_2$

Solution 1:

$$\frac{m_{NaNO_3}}{m_{NaNO_2}} = \frac{M_{NaNO_3}}{M_{NaNO_2}}$$ **(1,5p)**

$$m_{NaNO_3} = \frac{M_{NaNO_3}}{M_{NaNO_2}} \cdot m_{NaNO_2} = \frac{85.0\,{}^{g}\!/_{mol}}{69.0\,{}^{g}\!/_{mol}} \cdot 880g = 1084.06g \approx \underline{\underline{1080g}}$$ **(1,5p)**

Solution 2 :

NaNO$_3$ → NaNO$_2$ + ½O$_2$ (reaction equation not necessary, not required)

$$n_{NaNO_2} = \frac{m_{NaNO_2}}{M_{NaNO_2}} = \frac{880g}{69.0\,{}^{g}\!/_{mol}} = 12.7536\ mol$$ **(1p)**

$$n_{NaNO_3} = n_{NaNO_2}$$ **(1p)**

$$m_{NaNO_3} = n_{NaNO_3} \cdot M_{NaNO_3} = 12.7536 mol \cdot 85.0\,{}^{g}\!/_{mol} = 1084.06\ g \approx \underline{\underline{1080g}}$$ **(1p)**

9A) Use the ideal gas law $\frac{p\,V}{T} = constant$, in which T is in Kelvin units. First calculate the pressure in the tank when used outdoors. The tank is the same so the volume is constant.

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow p_2 = \frac{T_2\,p_1}{T_1} \; \frac{253.15 \cdot 50}{293.15} = 43.2 \text{ bar.}$$

The wrench needs 10 bar pressure, so the usable pressure is 43.2 – 10 = 33.2 bar. The volume of the air can be calculated based on the ideal gas law. The temperature is constant, p_1 is the pressure inside the tank and p_2 is the normal air pressure.

$$p_1\,V_1 = p_2\,V_2 \Rightarrow V_2 = \frac{V_1\,p_1}{p_2} = \frac{50 \cdot 33.2}{1} = 1660 \text{ L}$$

$$t = \frac{V_2}{V_{flow}} = \frac{1660}{200} = 8.3 \text{ min or 8 min.}$$

Grading:

Ideal gas law identified	**(1p)**
Correct idea of lower pressure when tank at cold	**(1p)**
Volume of air and time calculated	**(1p)**

9B) $V_{riverflow} = 1.80 m^3 / \text{min}$

$$c_{Ni} = 2.2 \, {}^{mg}\!/\!{}_{L}$$

$$m_{Ni,24h} = V_{riverflow} \cdot c_{Ni} = 1.80 \cdot 1000 \, {}^{L}\!/\!{}_{\text{min}} \cdot 2.2 \, {}^{mg}\!/\!{}_{L} \cdot 24 h/d \cdot 60 \, \text{min}/h$$

$$= 5702400 \, {}^{mg}\!/\!{}_{d} = 5702.4 \, {}^{g}\!/\!{}_{d}$$ **(1p)**

$$m_{Ni,80\%} = 0.80 \cdot m_{Ni,24h} = 4561.92 \, g$$ **(1p)**

$$c_{Ni} = \frac{m_{Ni,80\%}}{V_{lake}} = \frac{4561.92 \, g}{100000 m^3} = 0.04562 \, {}^{g}\!/\!{}_{m^3} = \underline{\underline{46 \, {}^{\mu g}\!/\!{}_{L}}}$$ **(1p)**

Limit of 20 µg/L was exceeded.

10A) There are four resistors in series in the circuit. One of the four resistors consists of four identical resistors in parallel. The parallel resistors yield to 1/4 R, so the total resistance is 3¼ R. If each R is equal to 40 Ω, the total resistance is

a) $R_{tot} = 3\frac{1}{4} \cdot 40\ \Omega = 130\ \Omega.$

b) From the Ohm's law we know

$V = R \cdot I$, so the current $I = V/R = 13/130 = 100$ mA.

Grading:

Understanding of the circuit: four resistors in series	**(1p)**
Correct R_{tot}, Ohm's law remembered	**(1p)**
Correct current	**(1p)**

10B) $m_{waste} = 1000\,000\ kg$

$m_{Cl} = 0.0062 \cdot 1000000\ kg = 6200\ kg$

$M_{Cl} = 35.5$
$M_{HCl} = 36.5$

Each Cl produces one HCl.

a) **Solution 1:**

$$\frac{m_{HCl}}{m_{Cl}} = \frac{M_{HCl}}{M_{Cl}}$$ **(0.5p)**

$$m_{HCl} = \frac{M_{HCl}}{M_{Cl}} \cdot m_{Cl} = \frac{36.5 \, {}^{g}\!/\!_{mol}}{35.5 \, {}^{g}\!/\!_{mol}} \cdot 6200kg = 6374.65kg \approx \underline{\underline{6400kg}}$$ **(1p)**

a) **Solution 2:**

$Cl_2 + H_2 \rightarrow 2HCl$ (reaction equation not necessary; not required)

$$n_{Cl} = \frac{m_{Cl}}{M_{Cl}} = \frac{6200kg}{35.5 \, {}^{g}\!/\!_{mol}} = 174.648mol \quad \text{OR} \quad n_{Cl_2} = \frac{m_{Cl_2}}{M_{Cl_2}} = \frac{6200kg}{71.0 \, {}^{g}\!/\!_{mol}} = 87.324mol$$ **(0.5p)**

$$n_{HCl} = n_{Cl} \qquad \text{OR} \qquad n_{HCl} = 2 \cdot n_{Cl_2}$$ **(0.5p)**

$$m_{HCl} = n_{HCl} \cdot M_{HCl} = 174.648mol \cdot 36.5 \, {}^{g}\!/\!_{mol} = 6374.65kg \approx \underline{\underline{6400\ kg}}$$ **(0.5p)**

b) $2\ HCl + CaO \rightarrow CaCl_2 + H_2O$ **(0.5p)**

$$n_{CaO} = ½ \cdot n_{HCl} = ½ \cdot n_{Cl} = ½ * \frac{m_{Cl}}{M_{Cl}} = 87.3239\ mol$$ **(0.5p)**

$$m_{CaO} = n_{CaO} \cdot M_{CaO} = 87.3239 mol \cdot 56.1 {}^{g}\!/\!{}_{mol} = 4898.9 kg \approx \underline{\underline{4900\ kg}}$$ **(0.5p)**